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# ANALYSIS ON 2 X 2 MATRIX WITH RESPECT TO VECTOR SPACE

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## ABSTRACT

This Plan gives the short view on finding the number of possible subspace for a 2x2 matrices as in the various of dependence and indepenence with respect to the real and complex field. Also it has generalized its dimension when it satisfies some conditions as sum of rows is zero, sum of columns is zero, sum of main diagonal is zero and sum of lateral diagonal is zero. Then it further developed by find the basis for those spaces. Mainly by 2x2 matrices it has the base to find the suduko in a generalized version in specification according to the grids.

**KEYWORDS :** Matrices, Dimension, Subspace

## INTRODUCTION

Vector Spaces as abstract algebraic entities were first defined by the Italian mathematician Giuseppe Peano in 1888. Peano called his vector spaces as linear systems because he correctly saw that one can obtain any vector in the space from a linear combination of finitely many vectors and scalars.

$$av + bw + \cdots + cz$$

Descartes and Fermat have first discussed the vector spaces R 2 and R 3 in which the way we are presented today, but the emphasis was on points and graphing rather than on the concept of avector. The notion of a vector is traceable to Bolzano, in its concrete form. The realization that abstract vector spaces abound in mathematics did not appear until the late nineteenth century. The modern definition seems to be due to the Italian mathematician Peano, who presented the modern form of the axioms of a Vector Space.

The focus on the important examples of function spaces as vector spaces is to be found in the work of Lebesgue and was formalized by Hilbert and Banach in the twentieth century.

In this project we are going to focus on the discussion about the dimensions of vector spaces and also conclude with the number of possible subspace of the vector space.

## DIMENSION

The number of vectors in a basis for V is called the dimension of V and is denotedby DIM(V).

Let V be a vector space. A minimal set of vectors in V that spans V is called a basis for V. Equivalently, a basis for V is a set of vectors that is linearly independent and spans V.

- $R_2$ over R has 3 dimension.
- $M_{2X2}$ [R] over R has 4 dinemsion.
- Complex over Real has 2 dimension.

**NUMBER OF POSSIBLE SUBSPACES FOR 2X2 MATRICS**

For 2x2 matrix

$$M_{2X2}[R] = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \, a, b, c, d \in R \right\}$$

$M_{2X2}[R]$ is avector space under R is true.

Then the basis be

$$B = \left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}\right\}$$

We all familiar that of a vector space we can form an infinite set of basis and for 2x2 the maximum posssibility of dimension is four (4).

Let us consider only one element form the space then

$$A = \left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}\right\} \quad \mathrm{B} = \left\{\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}\right\} \quad \mathrm{C} = \left\{ \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}, \right\} \quad \mathrm{D} = \left\{\begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}\right\}$$

For using only one element from the basis we get = 4 combinations.

We get 4 elements of one dimension.

Let us consider two elements form the basis then

$$E = \left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}\right\} \qquad \mathrm{F} = \left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}\right\}$$

$$G = \left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}\right\} \qquad H = \left\{\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}\right\}$$

$$I = \left\{\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}\right\} \qquad J = \left\{\begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}\right\}$$

For using two elements from the basis we get = 6 combinations.

Consider only E we will have two cases.

- case(i)

a ≠ b

- case(ii)

a + b = 0

For each set we can get one 1 dimension space and one 2 dimension space .

Now for the whole two sets, we can get

Six 1-dimension vector space.

Six 2-dimension vector space.

Let us consider three elements form the basis then

$$K = \left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}\right\}$$

$$L = \left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}\right\}$$

$$M = \left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}\right\}$$

$$N = \left\{\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix}\right\}$$

For using three elements from the basis we can get = 4 combination.

Consider only K we will have three cases.

- case(i)

a+b+c=0

• case(ii)
a independent b+c=0
b independent a+c=0
c independent a+b=0
• case(iii)
a,b,c where all are independent
For each set we can get one 1-dimension space , three 2-dimension space and one 3-dimension space .
Four 1-dimension vector space.
Twelve 2-dimension vector space.
Four 3-dimension vector space.
Let us consider four element form the basis then

$$O = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & d \end{bmatrix} \right\}$$

For using four elements from the basis we can get = 1 combination.
Consider O we will have five cases.
• case(i)
a+b+c+d=0
• case(ii)
a independent b+c+d=0
b independent a+c+d=0
c independent a+b+d=0
d independent a+b+c=0
• case(iii)
a+b=0 and c+d=0
a+c=0 and b+d=0
a+d=0 and b+c=0

• case (iv)
a,b independent c+d=0
a,c independent b+d=0
a,d independent b+c=0
b,c independent a+d=0
b,d independent a+c=0
c,d independent a+b=0
• case (v)
a,b,c,d independent
From each cases we can get one 1-dimension space , seven 2-dimension space , six 3-dimension space one 4-dimension space .

One 1-dimension vector space.
Seven 2-dimension vector space.
Six 3-dimension vector space.
One 4-dimension vector space.
From the above,we can get distinct

one dimension space = 15
Two dimension space = 25

Three dimension space = 10
Four dimension space = 1
Total we have 51 spaces

## CONCLUSION

From the above content we can conclude that the number of possible distinct subspaces for a 2x2 matrics is 51

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